

Chern-Simons AdS_5 supergravity in a Randall-Sundrum background

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Abstract

Chern-Simons AdS supergravity theories are gauge theories for the super-AdS group. These theories possess a fermionic symmetry which differs from standard supersymmetry. In this paper, we study five-dimensional Chern-Simons AdS supergravity in a Randall-Sundrum scenario with two Minkowski 3-branes. After making modifications to the $D = 5$ Chern-Simons AdS supergravity action and fermionic symmetry transformations, we obtain a \mathbb{Z}_2 -invariant total action $S = \tilde{S}_{bulk} + S_{brane}$ and fermionic transformations $\tilde{\delta}_\epsilon$. While $\tilde{\delta}_\epsilon \tilde{S}_{bulk} = 0$, the fermionic symmetry is broken by S_{brane} . Our total action reduces to the original Randall-Sundrum model when \tilde{S}_{bulk} is restricted to its gravitational sector. We solve the Killing spinor equations for a bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields.

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1 Introduction

Chern-Simons AdS supergravity [1, 2, 3] theories can be constructed only in odd spacetime dimensions. As the name implies, they are gauge theories for supersymmetric extensions of the AdS group.¹ They have a fiber bundle structure and hence are potentially renormalizable [2]. The dynamical fields form a single adS superalgebra-valued connection and hence the supersymmetry algebra closes automatically *off-shell* without requiring auxiliary fields [4]. The Lagrangian in dimension $D = 2n - 1$ is a Chern-Simons $(2n - 1)$ -form for the super-adS connection and is a polynomial of order n in the corresponding curvature. Unlike standard supergravity theories, there can be a mismatch between the number of bosonic and fermionic degrees of freedom.² For this reason, the ‘supersymmetry’ of Chern-Simons AdS supergravity theories is perhaps better referred to as a fermionic symmetry.

$D = 11$, $N = 1$ *Chern-Simons AdS supergravity* may correspond to an off-shell supergravity limit of M-theory [2, 3]. It has expected features of M-theory which are not shared by $D = 11$ *Cremmer-Julia-Scherk (CJS) supergravity* [5]. These features include an $osp(32|1)$ superalgebra [6] and higher powers of curvature [7]. *Hořava-Witten theory* [8] is obtained from CJS supergravity by compactifying on an S^1/\mathbb{Z}_2 orbifold and requiring gauge and gravitational anomalies to cancel. This theory gives the low energy, strongly coupled limit of the heterotic $E_8 \times E_8$ string theory. In light of the above discussion, it would be interesting to reformulate Hořava-Witten theory with $D = 11$, $N = 1$ Chern-Simons AdS supergravity.

Reformulating Hořava-Witten theory as described above may prove to be difficult. It is simpler to compactify the five-dimensional version of Chern-Simons AdS supergravity on an S^1/\mathbb{Z}_2 orbifold and ignore anomaly cancellation issues. Canonical sectors of $D = 5$ Chern-Simons AdS supergravity have been investigated in locally AdS_5 backgrounds possessing a spatial boundary with topology $S^1 \times S^1 \times S^1$ located at infinity [9]. In this paper, as a preamble to reformulating Hořava-Witten theory, we will study $D = 5$ Chern-Simons AdS supergravity in a Randall-Sundrum background with two Minkowski

¹ The AdS group in dimension $D \geq 2$ is $SO(D - 1, 2)$. The corresponding super-AdS groups are given in [3]. For $D = 5$ and $D = 11$, the super-AdS groups are respectively $SU(2, 2|N)$ and $OSP(32|N)$.

² For example, in $D = 5$ *Chern-Simons AdS supergravity* [1], the number of bosonic degrees of freedom ($N^2 + 15$) is equal to the number of fermionic degrees of freedom ($8N$) only for $N = 3$ and $N = 5$.

3-branes [10]. We choose coordinates $x^\mu = (x^{\bar{\mu}}, x^5)$ to parameterize the five-dimensional spacetime manifold.³ In terms of these coordinates, the background metric takes the form

$$g_{\mu\nu}dx^\mu dx^\nu = \mathfrak{a}^2(x^5)\eta_{\bar{\mu}\bar{\nu}}^{(4)}dx^{\bar{\mu}}dx^{\bar{\nu}} + (dx^5)^2, \quad (1.1)$$

where $\eta_{\bar{\mu}\bar{\nu}}^{(4)} = \text{diag}(-1, 1, 1, 1)_{\bar{\mu}\bar{\nu}}$, $\mathfrak{a}(x^5) \equiv \exp(-|x^5|/\ell)$ is the *warp factor*, and ℓ is the AdS_5 curvature radius. The coordinate x^5 parameterizes an S^1/\mathbb{Z}_2 orbifold, where the circle S^1 has radius ρ and \mathbb{Z}_2 acts as $x^5 \rightarrow -x^5$. We choose the range $-\pi\rho \leq x^5 \leq \pi\rho$ with the endpoints identified as $x^5 \simeq x^5 + 2\pi\rho$. The Minkowski 3-branes are located at the \mathbb{Z}_2 fixed points $x^5 = 0$ and $x^5 = \pi\rho$. These 3-branes have corresponding tensions $\mathcal{T}^{(0)}$ and $\mathcal{T}^{(\pi\rho)}$ and may support $(3+1)$ -dimensional field theories.

This paper is organized as follows: In Section 2, we construct a \mathbb{Z}_2 -invariant bulk theory. This bulk theory is obtained by making modifications to the $D=5$ Chern-Simons AdS supergravity action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed. The variation of the resulting bulk action S_{bulk} under the resulting fermionic transformations δ_ϵ vanishes everywhere except at the \mathbb{Z}_2 fixed points. We calculate $\delta_\epsilon S_{bulk}$ in Section 3. In Section 4, we modify S_{bulk} and δ_ϵ to obtain a modified \mathbb{Z}_2 -invariant bulk theory. The modified bulk action \tilde{S}_{bulk} is invariant under the modified fermionic transformations $\tilde{\delta}_\epsilon$. In Section 5, we complete our model by adding the brane action S_{brane} . We show in Section 6 that our total action

$$S = \tilde{S}_{bulk} + S_{brane} \quad (1.2)$$

reduces to the original Randall-Sundrum model [10] when \tilde{S}_{bulk} is restricted to its gravitational sector. In Section 7, we solve the Killing spinor equations for a purely bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields. Our concluding remarks are given in Section 8. Finally, in the Appendix, we work out the fünfbein, spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar for our metric (1.1).

³ We use indices $\mu, \nu, \dots = 0, 1, 2, 3, 5$ for local spacetime and $a, b, \dots = \dot{0}, \dot{1}, \dot{2}, \dot{3}, \dot{5}$ for tangent spacetime. The corresponding metrics, $g_{\mu\nu}$ and $\eta_{ab} = \text{diag}(-1, 1, 1, 1, 1)_{ab}$, are related by $g_{\mu\nu} = e_\mu^a e_\nu^b \eta_{ab}$, where e_μ^a is the fünfbein. Barred indices $\bar{\mu}, \bar{\nu}, \dots = 0, 1, 2, 3$, and $\bar{a}, \bar{b}, \dots = \dot{0}, \dot{1}, \dot{2}, \dot{3}$ denote the four-dimensional counterparts of μ, ν, \dots and a, b, \dots , respectively.

2 \mathbb{Z}_2 -invariant bulk theory

In this section, we construct a \mathbb{Z}_2 -invariant bulk theory. The bulk theory is obtained by making modifications to the $D = 5$ Chern-Simons AdS supergravity [1] action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed.

The field content of $D=5$ Chern-Simons AdS supergravity is the fünfbein e_μ^a , the spin connection ω_μ^{ab} , the $su(N)$ gauge connection $A_\mu = A_\mu^i \tau_i$, the $u(1)$ gauge connection B_μ , and N complex gravitini $\psi_{\mu r}$ which transform as Dirac spinors in a vector representation of $su(N)$.⁴ These fields form a connection for the adS superalgebra $su(2, 2|N)$. The action and fermionic symmetry transformations are given in [9] in terms of the AdS_5 curvature radius ℓ . The only free parameter in the action is a dimensionless constant k . To allow consistent \mathbb{Z}_2 orbifold conditions to be imposed, we make the following modifications:

1. Rescale the $su(N)$ and $u(1)$ gauge connections:

$$A \rightarrow g_A A, \quad B \rightarrow g_B B.$$

2. Replace g_A , g_B , ℓ^{-1} , and k by the \mathbb{Z}_2 -odd expressions⁵

$$\begin{aligned} G_A &\equiv g_A \operatorname{sgn}(x^5), & G_B &\equiv g_B \operatorname{sgn}(x^5), & L^{-1} &\equiv \ell^{-1} \operatorname{sgn}(x^5), \\ & & K &\equiv k \operatorname{sgn}(x^5). \end{aligned}$$

In this manner, we obtain the bulk action

$$S_{bulk} = S_{grav} + S_{su(N)} + S_{u(1)} + S_{ferm}, \quad (2.1)$$

⁴ We use indices $i, j, \dots = 1, \dots, N^2 - 1$ to label the $N \times N$ -dimensional $su(N)$ generators τ_i . The indices $r, s, \dots = 1, \dots, N$ label vector representations of $su(N)$. We will use the notation $A_s^r \equiv A^i(\tau_i)_s^r$. Spinor indices α, β, \dots will sometimes be suppressed.

⁵ The *signum function* $\operatorname{sgn}(x^5)$ is $+1$ for $0 < x^5 < \pi\rho$ and -1 for $-\pi\rho < x^5 < 0$. It obeys $\operatorname{sgn}^2(x^5) = 1$ and $\partial_5 \operatorname{sgn}(x^5) = 2[\delta(x^5) - \delta(x^5 - \pi\rho)]$.

where

$$\begin{aligned}
S_{grav} &= \int \frac{1}{8} K \varepsilon_{abcde} \left(\frac{1}{L} R^{ab} R^{cd} e^e + \frac{2}{3L^3} R^{ab} e^c e^d e^e + \frac{1}{5L^5} e^a e^b e^c e^d e^e \right), \\
S_{su(N)} &= \int iK \operatorname{str} \left(G_A^3 A F^2 - \frac{1}{2} G_A^4 A^3 F + \frac{1}{10} G_A^5 A^5 \right), \\
S_{u(1)} &= \int K \left[- \left(\frac{1}{4^2} - \frac{1}{N^2} \right) G_B^3 B (dB)^2 + \frac{3}{4L^2} (T^a T_a - \frac{L^2}{2} R^{ab} R_{ab} \right. \\
&\quad \left. - R^{ab} e_a e_b) G_B B - \frac{3}{N} G_A^2 G_B F_s^r F_r^s B \right], \\
S_{ferm} &= \int \frac{3}{2i} K \left(\bar{\psi}_\alpha^r \mathcal{R}_\beta^\alpha \nabla \psi_r^\beta + \bar{\psi}_\alpha^s \mathcal{F}_s^r \nabla \psi_r^\alpha \right) + c.c., \tag{2.2}
\end{aligned}$$

and the transformations

$$\begin{aligned}
\delta_\epsilon e^a &= -\frac{1}{2} (\bar{\psi}^r \Gamma^a \epsilon_r - \bar{\epsilon}^r \Gamma^a \psi_r), & \delta_\epsilon \omega^{ab} &= \frac{1}{4} (\bar{\psi}^r \Gamma^{ab} \epsilon_r - \bar{\epsilon}^r \Gamma^{ab} \psi_r), \\
\delta_\epsilon \psi_r &= -\nabla \epsilon_r, & \delta_\epsilon \bar{\psi}^r &= -\nabla \bar{\epsilon}^r, \\
\delta_\epsilon A_s^r &= i (\bar{\psi}^r \epsilon_s - \bar{\epsilon}^r \psi_s), & \delta_\epsilon B &= i (\bar{\psi}^r \epsilon_r - \bar{\epsilon}^r \psi_r). \tag{2.3}
\end{aligned}$$

In these expressions, Γ^a are the Dirac matrices⁶, $\Gamma^{ab} \equiv \frac{1}{2} (\Gamma^a \Gamma^b - \Gamma^b \Gamma^a)$, $R^{ab} = d\omega^{ab} + \omega^{ac} \omega_c^b$ is the curvature 2-form, $T^a = de^a + \omega^a_b e^b$ is the torsion 2-form, $F = dA + G_A A^2 = F^i \tau_i$ is the $su(N)$ curvature,

$$\begin{aligned}
\mathcal{R}_\beta^\alpha &\equiv \frac{1}{2L} T^a (\Gamma_a)_\beta^\alpha + \frac{1}{4} (R^{ab} + \frac{1}{L^2} e^a e^b) (\Gamma_{ab})_\beta^\alpha + \frac{i}{4} G_B dB \delta_\beta^\alpha - \frac{1}{2} \psi_s^\alpha \bar{\psi}_\beta^s, \\
\mathcal{F}_s^r &\equiv F_s^r + \frac{i}{N} G_B dB \delta_s^r - \frac{1}{2} \bar{\psi}_\beta^r \psi_s^\beta, \tag{2.4}
\end{aligned}$$

str is a symmetrized trace satisfying $\operatorname{str}(\tau_i \tau_j \tau_k) \equiv \frac{1}{2i} \operatorname{tr}(\{\tau_i, \tau_j\} \tau_k)$, ∇ is the $adS_5 \times su(N) \times u(1)$ covariant derivative, and

$$\begin{aligned}
\nabla \psi_r &\equiv \left(d + \frac{1}{4} \omega^{ab} \Gamma_{ab} + \frac{1}{2L} e^a \Gamma_a \right) \psi_r - G_A A_r^s \psi_s + i \left(\frac{1}{4} - \frac{1}{N} \right) G_B B \psi_r, \\
\nabla \epsilon_r &\equiv \left(d + \frac{1}{4} \omega^{ab} \Gamma_{ab} + \frac{1}{2L} e^a \Gamma_a \right) \epsilon_r - G_A A_r^s \epsilon_s + i \left(\frac{1}{4} - \frac{1}{N} \right) G_B B \epsilon_r. \tag{2.5}
\end{aligned}$$

Note that the results in the Appendix can be used to show that the torsion vanishes for our metric.

We impose the following orbifold conditions:

⁶ We choose a chiral basis for the 4×4 Dirac matrices

$$\Gamma^a = (\Gamma^{\bar{a}}, \Gamma^{\dot{5}}) = \left(\begin{bmatrix} \mathbf{0} & -i\sigma^{\bar{a}} \\ -i\bar{\sigma}^{\bar{a}} & \mathbf{0} \end{bmatrix}, \begin{bmatrix} -\mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \right),$$

where $\sigma^{\bar{a}} = (\mathbf{1}, \vec{\sigma})$ and $\bar{\sigma}^{\bar{a}} = (\mathbf{1}, -\vec{\sigma})$. These matrices satisfy $\operatorname{tr}(\Gamma_a \Gamma_b \Gamma_c \Gamma_d \Gamma_e) = -4i \varepsilon_{abcde}$, where ε_{abcde} is the Levi-Civita tensor and $\varepsilon^{\dot{0}\dot{1}\dot{2}\dot{3}\dot{5}} = 1$.

1. *Periodicity on S^1 .* The fields and the fermionic parameters ϵ_r , denoted generically by ϕ , are required to be periodic on the circle S^1 . That is,

$$\phi(x^{\bar{\mu}}, x^5) = \phi(x^{\bar{\mu}}, x^5 + 2\pi\rho). \quad (2.6)$$

2. *\mathbb{Z}_2 parity assignments.* The bosonic field components

$$\Phi = e_{\mu}^{\bar{a}}, e_5^{\dot{5}}, A_5^i, B_5, \quad \Theta = e_{\bar{\mu}}^{\dot{5}}, e_5^{\bar{a}}, A_{\bar{\mu}}^i, B_{\bar{\mu}}$$

are chosen to satisfy

$$\Phi(x^{\mu}, x^5) = +\Phi(x^{\mu}, -x^5), \quad \Theta(x^{\mu}, x^5) = -\Theta(x^{\mu}, -x^5). \quad (2.7)$$

That is, the Φ components are \mathbb{Z}_2 -even and the Θ components are \mathbb{Z}_2 -odd. For the gravitini, we require

$$\begin{aligned} \Gamma^{\dot{5}} \psi_{\bar{\mu}r}(x^{\bar{\mu}}, x^5) &= +\psi_{\bar{\mu}r}(x^{\bar{\mu}}, -x^5), \\ \Gamma^{\dot{5}} \psi_{5r}(x^{\bar{\mu}}, x^5) &= -\psi_{5r}(x^{\bar{\mu}}, -x^5). \end{aligned} \quad (2.8)$$

Finally, the fermionic parameters ϵ_r are required to satisfy

$$\Gamma^{\dot{5}} \epsilon_r(x^{\bar{\mu}}, x^5) = +\epsilon_r(x^{\mu}, -x^5). \quad (2.9)$$

These conditions imply that the \mathbb{Z}_2 -odd quantities vanish at the orbifold fixed points. It is straightforward to check that S_{bulk} is \mathbb{Z}_2 -even and that the transformations (2.3) are consistent with the \mathbb{Z}_2 parity assignments.

3 Calculation of $\delta_{\epsilon} S_{bulk}$

The $D = 5$ Chern-Simons AdS supergravity action is invariant (up to a boundary term) under its fermionic symmetry transformations. In Section 2, we modified this action and its fermionic transformations to obtain a \mathbb{Z}_2 -invariant bulk theory. Due to the signum functions introduced by the modifications, $\delta_{\epsilon} S_{bulk}$ contains terms which have no counterpart in the unmodified theory. More specifically, the extra terms arise from ∂_5 acting on

the signum functions to yield delta functions. Such ‘delta function’ contributions to $\delta_\epsilon S_{bulk}$ can potentially spoil the fermionic symmetry only at the \mathbb{Z}_2 fixed points. Thus, S_{bulk} is invariant under its fermionic transformations everywhere except perhaps at the \mathbb{Z}_2 fixed points. In this section, we will calculate $\delta_\epsilon S_{bulk}$.

For our metric and \mathbb{Z}_2 parity assignments, the uncanceled variation $\delta_\epsilon S_{bulk}$ arises from the variation of the 4-Fermi terms. The 4-Fermi terms are

$$\begin{aligned}
S_{\psi^4} &= \frac{3i}{4} \int K \left(\bar{\psi}_\alpha^r \psi_s^\alpha \bar{\psi}_\beta^s \nabla \psi_r^\beta + \bar{\psi}_\alpha^s \bar{\psi}_\beta^r \psi_s^\beta \nabla \psi_r^\alpha \right) + c.c. \\
&= \frac{3i}{2} \int K \bar{\psi}_\alpha^r \psi_s^\alpha \bar{\psi}_\beta^s \nabla \psi_r^\beta + c.c. \\
&= \frac{3i}{2} \int K \left(\bar{\psi}^r \psi_s \right) \left(\bar{\psi}^s \nabla \psi_r \right) + c.c. \\
&= \frac{3i}{2} \frac{1}{5!} \int d^5 x \varepsilon^{\mu\nu\rho\sigma\lambda} K \left(\bar{\psi}_\mu^r \psi_{\nu s} \right) \left(\bar{\psi}_\rho^s \nabla_\sigma \psi_{\lambda r} \right) + c.c. \\
&= \frac{3i}{2} \frac{1}{4!} \int d^5 x K \left[\varepsilon^{5\bar{\nu}\bar{\rho}\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}_5^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \nabla_{\bar{\sigma}} \psi_{\bar{\lambda} r} \right) + \varepsilon^{\bar{\mu}5\bar{\rho}\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}_{\bar{\mu}}^r \psi_{5s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \nabla_{\bar{\sigma}} \psi_{\bar{\lambda} r} \right) \right. \\
&\quad + \varepsilon^{\bar{\mu}\bar{\nu}5\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_5^s \nabla_{\bar{\sigma}} \psi_{\bar{\lambda} r} \right) + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \nabla_5 \psi_{\bar{\lambda} r} \right) \\
&\quad \left. + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}5} \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \nabla_{\bar{\sigma}} \psi_{5r} \right) \right] + c.c. \tag{3.1}
\end{aligned}$$

Let us now compute $\delta_\epsilon S_{bulk}$ by applying δ_ϵ to (3.1) and dropping all terms which contribute no delta functions. For our metric and \mathbb{Z}_2 parity assignments, we can drop all but

1. The ∂_μ part of ∇_μ .
2. The $-\partial_\mu \epsilon_r$ part of $\delta_\epsilon = -\nabla_\mu \epsilon_r$.

The only contributing terms are thus contained in the expression

$$\begin{aligned}
Q &\equiv -\frac{3i}{2} \frac{1}{4!} \int d^5 x K \left\{ \varepsilon^{5\bar{\nu}\bar{\rho}\bar{\sigma}\bar{\lambda}} \left(\partial_5 \bar{\epsilon}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda} r} \right) \right. \\
&\quad + \varepsilon^{\bar{\mu}5\bar{\rho}\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}_{\bar{\mu}}^r \partial_5 \epsilon_s \right) \left(\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda} r} \right) + \varepsilon^{\bar{\mu}\bar{\nu}5\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\partial_5 \bar{\epsilon}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda} r} \right) \\
&\quad + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} \left[\left(\partial_{\bar{\mu}} \bar{\epsilon}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \partial_5 \psi_{\bar{\lambda} r} \right) + \left(\bar{\psi}_{\bar{\mu}}^r \partial_{\bar{\nu}} \epsilon_s \right) \left(\bar{\psi}_{\bar{\rho}}^s \partial_5 \psi_{\bar{\lambda} r} \right) \right. \\
&\quad \left. + \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\partial_{\bar{\rho}} \bar{\epsilon}^s \partial_5 \psi_{\bar{\lambda} r} \right) + \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \partial_5 \partial_{\bar{\lambda}} \epsilon_r \right) \right] \\
&\quad \left. + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}5} \left(\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu} s} \right) \left(\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \partial_5 \epsilon_r \right) \right\} + c.c. \tag{3.2}
\end{aligned}$$

More specifically, the delta function terms contained in Q are obtained by integrating by parts and keeping only the terms in which ∂_5 acts on K . Thus,

$$\begin{aligned} \delta_\epsilon S_{bulk} = & \frac{3i}{2} \frac{1}{4!} \int d^5x \partial_5 K \left\{ \varepsilon^{5\bar{\nu}\bar{\rho}\bar{\sigma}\bar{\lambda}} (\bar{\epsilon}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}) \right. \\ & + \varepsilon^{\bar{\mu}5\bar{\rho}\bar{\sigma}\bar{\lambda}} (\bar{\psi}_{\bar{\mu}}^r \epsilon_s) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}) + \varepsilon^{\bar{\mu}\bar{\nu}5\bar{\sigma}\bar{\lambda}} (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\epsilon}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}) \\ & + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} \left[(\partial_{\bar{\mu}} \bar{\epsilon}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \psi_{\bar{\lambda}r}) + (\bar{\psi}_{\bar{\mu}}^r \partial_{\bar{\nu}} \epsilon_s) (\bar{\psi}_{\bar{\rho}}^s \psi_{\bar{\lambda}r}) \right. \\ & \quad \left. + (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\partial_{\bar{\rho}} \bar{\epsilon}^s \psi_{\bar{\lambda}r}) + (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\lambda}} \epsilon_r) \right] \\ & \left. + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}5} (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \epsilon_r) \right\} + c.c., \end{aligned} \quad (3.3)$$

where

$$\partial_5 K = 2k [\delta(x^5) - \delta(x^5 - \pi\rho)]. \quad (3.4)$$

4 Modified \mathbb{Z}_2 -invariant bulk theory

The result (3.3) for $\delta_\epsilon S_{bulk}$ demonstrates that S_{bulk} is not invariant under the fermionic transformations δ_ϵ . In this section, we will modify S_{bulk} and δ_ϵ by replacing the $adS_5 \times su(N) \times u(1)$ covariant derivative ∇ with $\tilde{\nabla}$, where

$$\begin{aligned} \tilde{\nabla}_\sigma \psi_{\lambda r} &\equiv \nabla_\sigma \psi_{\lambda r} + 2\delta_\sigma^5 \delta_\lambda^{\bar{\lambda}} [\delta(x^5) - \delta(x^5 - \pi\rho)] \text{sgn}(x^5) \Gamma_{\bar{\lambda}} \psi_{\bar{\lambda}r}, \\ \tilde{\nabla}_\sigma \epsilon_r &\equiv \nabla_\sigma \epsilon_r + 2\delta_\sigma^5 [\delta(x^5) - \delta(x^5 - \pi\rho)] \text{sgn}(x^5) \Gamma_{\bar{5}} \epsilon_r. \end{aligned} \quad (4.1)$$

We will show that the modified bulk action

$$\tilde{S}_{bulk} \equiv S_{bulk}(\nabla \rightarrow \tilde{\nabla}) \equiv S_{bulk} + \Delta S_{bulk} \quad (4.2)$$

is invariant under the modified transformations

$$\tilde{\delta}_\epsilon \equiv \delta_\epsilon(\nabla \rightarrow \tilde{\nabla}) \equiv \delta_\epsilon + \Delta\delta_\epsilon. \quad (4.3)$$

That is, we will show that

$$\tilde{\delta}_\epsilon \tilde{S}_{bulk} = \delta_\epsilon S_{bulk} + (\Delta\delta_\epsilon) S_{bulk} + \tilde{\delta}_\epsilon (\Delta S_{bulk}) \quad (4.4)$$

vanishes. It is straightforward to check that \tilde{S}_{bulk} is \mathbb{Z}_2 -invariant and the transformations $\tilde{\delta}_\epsilon$ are consistent with our \mathbb{Z}_2 parity assignments.

We begin by computing $(\Delta\delta_\epsilon) S_{bulk}$. For our metric and \mathbb{Z}_2 parity assignments, the only part of S_{bulk} which is not invariant under $\Delta\delta_\epsilon$ is S_{ψ^4} (given by (3.1)). Note that

$$(\Delta\delta_\epsilon) \psi_{\lambda r} = -2\delta_\lambda^5 [\delta(x^5) - \delta(x^5 - \pi\rho)] \text{sgn}(x^5) \Gamma_{\dot{5}} \epsilon_r. \quad (4.5)$$

Thus, after using $K \text{sgn}(x^5) = k$, (2.9), and (3.4), we obtain

$$\begin{aligned} (\Delta\delta_\epsilon) S_{bulk} = & -\frac{3i}{2} \frac{1}{4!} \int d^5x \partial_5 K \left[\varepsilon^{5\bar{\nu}\bar{\rho}\bar{\sigma}\bar{\lambda}} (\bar{\epsilon}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}) \right. \\ & + \varepsilon^{\bar{\mu}5\bar{\rho}\bar{\sigma}\bar{\lambda}} (\bar{\psi}_{\bar{\mu}}^r \epsilon_s) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}) + \varepsilon^{\bar{\mu}\bar{\nu}5\bar{\sigma}\bar{\lambda}} (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\epsilon}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}) \\ & \left. + \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}5} (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\sigma}} \epsilon_r) \right] + c.c. \end{aligned} \quad (4.6)$$

Now, let us compute $\tilde{\delta}_\epsilon (\Delta S_{bulk})$. For our metric and \mathbb{Z}_2 parity assignments, the only part of S_{bulk} which is changed by the replacement $\nabla \rightarrow \tilde{\nabla}$ is S_{ψ^4} . After using $K \text{sgn}(x^5) = k$, (2.8), and (3.4), we obtain

$$\Delta S_{bulk} = \frac{3i}{2} \frac{1}{4!} \int d^5x \partial_5 K \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \psi_{\bar{\lambda}r}) + c.c. \quad (4.7)$$

Applying $\tilde{\delta}_\epsilon$ to (4.7) yields

$$\begin{aligned} \tilde{\delta}_\epsilon (\Delta S_{bulk}) = & -\frac{3i}{2} \frac{1}{4!} \int d^5x \partial_5 K \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} \left[(\partial_{\bar{\mu}} \bar{\epsilon}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \psi_{\bar{\lambda}r}) + (\bar{\psi}_{\bar{\mu}}^r \partial_{\bar{\nu}} \epsilon_s) (\bar{\psi}_{\bar{\rho}}^s \psi_{\bar{\lambda}r}) \right. \\ & \left. + (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\partial_{\bar{\rho}} \bar{\epsilon}^s \psi_{\bar{\lambda}r}) + (\bar{\psi}_{\bar{\mu}}^r \psi_{\bar{\nu}s}) (\bar{\psi}_{\bar{\rho}}^s \partial_{\bar{\lambda}} \epsilon_r) \right] + c.c. \end{aligned} \quad (4.8)$$

Using the results (3.3), (4.6), and (4.8) in (4.4) yields

$$\tilde{\delta}_\epsilon \tilde{S}_{bulk} = 0. \quad (4.9)$$

5 Brane action

To complete our model, we add the brane action S_{brane} . In the absence of particle excitations, the brane action consists of brane tensions. That is,

$$S_{brane} = - \int d^5x e^{(4)} [\mathcal{T}^{(0)} \delta(x^5) + \mathcal{T}^{(\pi\rho)} \delta(x^5 - \pi\rho)] + \text{excitations}, \quad (5.1)$$

where $e^{(4)} \equiv \det(e_{\bar{\mu}}^{\bar{a}})$. As discussed in Section 2, \mathbb{Z}_2 -odd quantities vanish at the \mathbb{Z}_2 fixed points. Thus, it is clear that S_{brane} is \mathbb{Z}_2 -even. Further discussion of 3-brane actions can be found in [11].

6 Connection with original RS model

In this section, we will show that our total action $S = \tilde{S}_{bulk} + S_{brane}$ reduces to the original Randall-Sundrum model [10] when \tilde{S}_{bulk} is restricted to its gravitational sector.

The gravitational sector of \tilde{S}_{bulk} is S_{grav} , given by the first equation of (2.2). S_{grav} consists of three terms:

1. The ‘Gauss-Bonnet’ term $\int \frac{1}{8} K \varepsilon_{abcde} R^{ab} R^{cd} e^e / L$.
2. The ‘Einstein-Hilbert’ term $\int \frac{1}{8} \cdot \frac{2}{3} K \varepsilon_{abcde} R^{ab} e^c e^d e^e / L^3$.
3. The ‘cosmological constant’ term $\int \frac{1}{8} \cdot \frac{1}{5} K \varepsilon_{abcde} e^a e^b e^c e^d e^e / L^5$.

For our metric, the first term can be expressed as a linear combination of the other two. Summing the three terms yields an effective Einstein-Hilbert term and an effective cosmological constant term. To demonstrate this explicitly, let us evaluate S_{grav} for our metric. Using the results in the Appendix, we obtain

$$\begin{aligned} \varepsilon_{abcde} R^{ab} R^{cd} e^e &= d^5 x \, e \left(-\frac{120}{\ell^4} + \frac{192}{\ell^3} [\delta(x^5) - \delta(x^5 - \pi\rho)] \right), \\ \varepsilon_{abcde} R^{ab} e^c e^d e^e &= d^5 x \, e (-6R), \\ \varepsilon_{abcde} e^a e^b e^c e^d e^e &= d^5 x \, e (-5!), \end{aligned} \tag{6.1}$$

where $e \equiv \det(e_\mu^a)$. Thus,

$$\begin{aligned} S_{grav} &= \int d^5 x \, e \frac{1}{8} \left\{ \frac{k}{\ell} \left(-\frac{120}{\ell^4} + \frac{192}{\ell^3} [\delta(x^5) - \delta(x^5 - \pi\rho)] \right) \right. \\ &\quad \left. + \frac{2k}{3\ell^3} (-6R) + \frac{k}{5\ell^5} (-5!) \right\} \\ &= \int d^5 x \, e \frac{k}{\ell^3} \left\{ -\frac{15}{\ell^2} + \frac{24}{\ell} [\delta(x^5) - \delta(x^5 - \pi\rho)] - \frac{1}{2}R - \frac{3}{\ell^2} \right\} \\ &= \int d^5 x \, e \frac{k}{\ell^3} \left\{ \frac{3}{2} \left(-\frac{20}{\ell^2} + \frac{16}{\ell} [\delta(x^5) - \delta(x^5 - \pi\rho)] \right) - \frac{1}{2}R + \frac{12}{\ell^2} \right\} \\ &= \int d^5 x \, e \frac{k}{\ell^3} \left(R + \frac{12}{\ell^2} \right) \\ &= \int d^5 x \, e (2M^3 R - \Lambda), \end{aligned} \tag{6.2}$$

where M is the five-dimensional gravitational mass scale⁷, Λ is the bulk cosmological constant, and

$$M^3 = \frac{k}{2\ell^3}, \quad \Lambda = -\frac{24M^3}{\ell^2}. \quad (6.3)$$

Combining the result (6.2) with (5.1), we obtain the action of the original Randall-Sundrum model. It is shown in [10] that the five-dimensional vacuum Einstein's equations for this system,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{4M^3} \left\{ g_{\mu\nu}\Lambda + \frac{e^{(4)}}{e} \delta_{\mu}^{\bar{\mu}} \delta_{\nu}^{\bar{\nu}} g_{\bar{\mu}\bar{\nu}} [\mathcal{T}^{(0)}\delta(x^5) + \mathcal{T}^{(\pi\rho)}\delta(x^5 - \pi\rho)] \right\}, \quad (6.4)$$

are solved by our metric provided that the relations

$$\mathcal{T}^{(0)} = -\mathcal{T}^{(\pi\rho)} = 24M^3/\ell, \quad \Lambda = -24M^3/\ell^2 \quad (6.5)$$

are satisfied.

7 Killing spinors

In this section, we will solve the Killing spinor equations for a purely bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields. In this case, the Killing spinor equations reduce to

$$\begin{aligned} 0 &= \delta_{\epsilon} \psi_{\bar{\mu}r} = -\partial_{\bar{\mu}} \epsilon_r - \frac{1}{2} \frac{\mathbf{a}'}{\mathbf{a}} \Gamma_{\bar{\mu}} (\Gamma_{\dot{5}} - 1) \epsilon_r, \\ 0 &= \delta_{\epsilon} \psi_{5r} = -\partial_5 \epsilon_r + \frac{1}{2} \frac{\mathbf{a}'}{\mathbf{a}} \Gamma_{\dot{5}} \epsilon_r - 2 [\delta(x^5) - \delta(x^5 - \pi\rho)] \text{sgn}(x^5) \Gamma_{\dot{5}} \epsilon_r. \end{aligned} \quad (7.1)$$

To solve these equations, split ϵ_r into \mathbb{Z}_2 -even (ϵ_r^+) and \mathbb{Z}_2 -odd (ϵ_r^-) pieces:

$$\epsilon_r = \epsilon_r^+ + \epsilon_r^-, \quad (7.2)$$

where

$$\epsilon_r^{\pm} \equiv \frac{1}{2} (\epsilon_r \pm \Gamma_{\dot{5}} \epsilon_r) = \pm \Gamma_{\dot{5}} \epsilon_r^{\pm}. \quad (7.3)$$

⁷ M is related to the four-dimensional gravitational mass scale $M_{(4)} = 2.43 \times 10^{18}$ GeV by $M_{(4)}^2 = M^3 \int_{-\pi\rho}^{+\pi\rho} dx^5 \mathbf{a}^2(x^5) = M^3 \ell [1 - \exp(-2\pi\rho/\ell)]$. The effective mass scales on the 3-branes at $x^5 = 0$ and $x^5 = \pi\rho$ are respectively $M_{(4)}$ and $M_{(4)} e^{-\pi\rho/\ell}$. If the Standard Model fields live on the 3-brane at $x^5 = \pi\rho$, then $M_{(4)} e^{-\pi\rho/\ell}$ can be associated with the electroweak scale.

We obtain the following system of equations:

$$\begin{aligned}
\partial_{\bar{\mu}} \epsilon_r^+ &= -(\mathfrak{a}'/\mathfrak{a}) \Gamma_{\bar{\mu}} \Gamma_{\dot{5}} \epsilon_r^-, \\
\partial_{\bar{\mu}} \epsilon_r^- &= 0, \\
\partial_5 \epsilon_r^+ &= +\frac{1}{2} (\mathfrak{a}'/\mathfrak{a}) \epsilon_r^+ - 2 [\delta(x^5) - \delta(x^5 - \pi\rho)] \text{sgn}(x^5) \epsilon_r^+, \\
\partial_5 \epsilon_r^- &= -\frac{1}{2} (\mathfrak{a}'/\mathfrak{a}) \epsilon_r^- + 2 [\delta(x^5) - \delta(x^5 - \pi\rho)] \text{sgn}(x^5) \epsilon_r^-.
\end{aligned} \tag{7.4}$$

These equations are solved by

$$\begin{aligned}
\epsilon_r^+ &= \mathfrak{a}^{1/2} \left[-(\mathfrak{a}'/\mathfrak{a}^2) x^{\bar{\mu}} \Gamma_{\bar{\mu}} \Gamma_{\dot{5}} \text{sgn}(x^5) \chi_r^{-(0)} + \chi_r^{+(0)} \right] \\
&= \mathfrak{a}^{1/2} \left[(1/\ell) x^{\bar{\mu}} \delta_{\bar{\mu}}^{\bar{a}} \Gamma_{\bar{a}} \Gamma_{\dot{5}} \chi_r^{-(0)} + \chi_r^{+(0)} \right], \\
\epsilon_r^- &= \mathfrak{a}^{-1/2} \text{sgn}(x^5) \chi_r^{-(0)},
\end{aligned} \tag{7.5}$$

where $\chi_r^{+(0)}$ and $\chi_r^{-(0)}$ are constant (projected) spinors.⁸ Thus, our solution for the Killing spinors is

$$\epsilon_r = \mathfrak{a}^{1/2} \chi_r^{+(0)} + \mathfrak{a}^{-1/2} \text{sgn}(x^5) \left(1 - \frac{\mathfrak{a}'}{\mathfrak{a}} x^{\bar{\mu}} \Gamma_{\bar{\mu}} \Gamma_{\dot{5}} \right) \chi_r^{-(0)}. \tag{7.6}$$

8 Conclusion

We have constructed a Randall-Sundrum scenario from $D=5$ Chern-Simons AdS supergravity. Our total action $S = \tilde{S}_{bulk} + S_{brane}$ is \mathbb{Z}_2 -invariant. \tilde{S}_{bulk} is invariant under the fermionic transformations $\tilde{\delta}_\epsilon$. However,

$$\tilde{\delta}_\epsilon S_{brane} = - \int d^5 x \tilde{\delta}_\epsilon e^{(4)} \left[\mathcal{T}^{(0)} \delta(x^5) + \mathcal{T}^{(\pi\rho)} \delta(x^5 - \pi\rho) \right] + \dots, \tag{8.1}$$

where

$$\tilde{\delta}_\epsilon e^{(4)} = e^{(4)} \left[-\frac{1}{2} (\bar{\psi}_{\bar{\mu}}^r \Gamma^{\bar{\mu}} \epsilon_r - \bar{\epsilon}^r \Gamma^{\bar{\mu}} \psi_{\bar{\mu}r}) \right]. \tag{8.2}$$

Thus, the fermionic symmetry is broken by S_{brane} . Nevertheless, the Killing spinors of Section 7 are globally defined.

⁸ It is straightforward to check that (7.5) satisfies the first, second, and fourth equations of (7.4). There is, however, a subtlety in checking that (7.5) satisfies the third equation of (7.4). Unlike ϵ_r^- , ϵ_r^+ is a smooth function of x^5 . Thus, the second term on the right side of the third equation of (7.4) contributes nothing.

Our model reduces to the original Randall-Sundrum model [10] when \tilde{S}_{bulk} is restricted to its gravitational sector. The original Randall-Sundrum model involves the fine-tuning relations

$$\mathcal{T}^{(0)} = -\mathcal{T}^{(\pi\rho)} = 24M^3/\ell, \quad \Lambda = -24M^3/\ell^2.$$

Randall-Sundrum scenarios constructed from standard $D = 5$ supergravity theories yield these relations (up to an overall normalization factor) as a consequence of local supersymmetry (some examples are given in [12]). In our case, the relation $\Lambda = -24M^3/\ell^2$ follows from our metric choice. We do not obtain the relations $\mathcal{T}^{(0)} = -\mathcal{T}^{(\pi\rho)} = 24M^3/\ell$ as a consequence of local fermionic symmetry. These are fine-tuning relations in our model.

A Appendix

In this Appendix, we work out the fünfbein, spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar for our metric (1.1). For the fünfbein, we obtain

$$\begin{aligned} e_{\bar{\mu}}^{\bar{a}} &= \mathbf{a} \delta_{\bar{\mu}}^{\bar{a}}, & e_{\bar{\mu}\bar{a}} &= e_{\bar{\mu}}^{\bar{b}} \eta_{\bar{b}\bar{a}}, & e^{\bar{\mu}\bar{a}} &= g^{\bar{\mu}\bar{\nu}} e_{\bar{\nu}}^{\bar{a}}, \\ e_{\bar{a}}^{\bar{\mu}} &= \mathbf{a}^{-1} \delta_{\bar{a}}^{\bar{\mu}}, & e_{\bar{a}\bar{\mu}} &= e_{\bar{a}}^{\bar{\nu}} g_{\bar{\nu}\bar{\mu}}, & e^{\bar{a}\bar{\mu}} &= \eta^{\bar{a}\bar{b}} e_{\bar{b}}^{\bar{\mu}}, \\ e_{\dot{5}}^{\dot{5}} &= e_{\dot{5}\dot{5}} = e^{5\dot{5}} = 1, & e_{\dot{5}}^5 &= e_{\dot{5}5} = e^{\dot{5}5} = 1. \end{aligned} \quad (\text{A.1})$$

Our conventions for the spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar are respectively

$$\begin{aligned} \omega_{\mu}^{ab} &= \frac{1}{2} e^{\nu a} (\partial_{\mu} e_{\nu}^b - \partial_{\nu} e_{\mu}^b) - \frac{1}{2} e^{\nu b} (\partial_{\mu} e_{\nu}^a - \partial_{\nu} e_{\mu}^a) \\ &\quad - \frac{1}{2} e^{\rho a} e^{\sigma b} (\partial_{\rho} e_{\sigma c} - \partial_{\sigma} e_{\rho c}) e_{\mu}^c, \\ R_{\mu\nu}^{ab} &= \partial_{\mu} \omega_{\nu}^{ab} - \partial_{\nu} \omega_{\mu}^{ab} + \omega_{\mu}^{ac} \omega_{\nu c}^b - \omega_{\nu}^{ac} \omega_{\mu c}^b, \\ R_{\nu\sigma} &= R_{\mu\nu}^{ab} e_a^{\mu} e_{b\sigma}, \quad R = e_a^{\mu} e_b^{\nu} R_{\mu\nu}^{ab}. \end{aligned}$$

For the metric (1.1), the nonzero quantities here are

$$\omega_{\bar{\mu}}^{\bar{a}\dot{5}} = -\omega_{\bar{\mu}}^{\dot{5}\bar{a}} = \mathbf{a}' \delta_{\bar{\mu}}^{\bar{a}} = -e_{\bar{\mu}}^{\bar{a}}/L, \quad (\text{A.2})$$

$$\begin{aligned} R_{\bar{\mu}\bar{\nu}}^{\bar{a}\bar{b}} &= -\mathbf{a}'^2 (\delta_{\bar{\mu}}^{\bar{a}} \delta_{\bar{\nu}}^{\bar{b}} - \delta_{\bar{\mu}}^{\bar{b}} \delta_{\bar{\nu}}^{\bar{a}}) = -(e_{\bar{\mu}}^{\bar{a}} e_{\bar{\nu}}^{\bar{b}} - e_{\bar{\mu}}^{\bar{b}} e_{\bar{\nu}}^{\bar{a}})/\ell^2, \\ R_{5\bar{\mu}}^{\bar{a}\dot{5}} &= \mathbf{a}'' \delta_{\bar{\mu}}^{\bar{a}} = e_{\bar{\mu}}^{\bar{a}} \{1/\ell^2 - 2[\delta(x^5) - \delta(x^5 - \pi\rho)]/\ell\}, \end{aligned} \quad (\text{A.3})$$

$$R_{\bar{\mu}\bar{\nu}} = -(\mathbf{a}\mathbf{a}'' + 3\mathbf{a}'^2)\eta_{\bar{\mu}\bar{\nu}} = -\{4/\ell^2 - 2[\delta(x^5) - \delta(x^5 - \pi\rho)]/\ell\} g_{\bar{\mu}\bar{\nu}},$$

$$R_{55} = -4\mathbf{a}^{-1}\mathbf{a}'' = -\{4/\ell^2 - 8[\delta(x^5) - \delta(x^5 - \pi\rho)]/\ell\}, \quad (\text{A.4})$$

$$R = -8\mathbf{a}^{-1}\mathbf{a}'' - 12\mathbf{a}^{-2}\mathbf{a}'^2 = -20/\ell^2 + 16[\delta(x^5) - \delta(x^5 - \pi\rho)]/\ell, \quad (\text{A.5})$$

and those related to (A.3) by $R_{\mu\nu}{}^{ab} = -R_{\nu\mu}{}^{ab} = -R_{\mu\nu}{}^{ba}$. The prime symbol $'$ denotes partial differentiation with respect to x^5 .

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